
2nd year Geometry and topology exam May-Jun 1996

Do not write on both sides of the invigilator

1. Say what is meant by a motion of Euclidean n -space $\mathbb{E}^n$. Explain without detailed proof how a motion of $\mathbb{E}^n$ can be represented in matrix terms. If $n = 2$, show how do to count parameters in this representation.

In these terms,

- (i) write down all motions of $\mathbb{E}^2$ that fix $(0, 0) \in \mathbb{E}^2$ and take $(3, 4)$ to $(5, 0)$;
- (ii) write down all motions of $\mathbb{E}^2$ taking

$$(-1, -1) \mapsto (1, 2) \quad \text{and} \quad (1, 2) \mapsto (6, 2).$$

2. Define projective space $\mathbb{P}^n$ and the notions of linear subspace and linear span. State and prove the formula for the dimension of intersection, explaining carefully any special conventions used. (Results on vector spaces may be used freely.) What is the advantage of using projective space for this result?

Let L_1, L_2 and m be 3 lines of $\mathbb{P}^3$ such that

$$L_1 \cap m = \emptyset, \quad L_2 \cap m = \emptyset \quad \text{and} \quad L_1 \cap L_2 = \{P\}$$

(one point). Find all the lines of $\mathbb{P}^3$ meeting L_1, L_2 and m .

4. Let $\triangle PQR$ be a spherical right-angled triangle with right angle at P . State and prove from first principles a formula expressing the hypotenuse $d(Q, R)$ in terms of the other two sides $d(P, Q)$ and $d(P, R)$. (All lengths are measured in terms of the intrinsic spherical metric of S^2 .) Verify that your formula approximates Pythagoras' theorem when the triangle is small compared with the radius of the sphere.

(There's no credit in this question for writing down without proof general formulas of spherical trigonometry, although these may help organise your thoughts.)
