MA243 Geometry

Worksheet 5

Deadline: Thu 4th Nov 2004, 2:00pm.
Please staple your solutions.

A. **Example of a glide**

Express the map $f\colon (x,y) \mapsto (x+3, -y-4)$ on $\mathbb{E}^2$ as a glide reflection.

B. **Walking a wardrobe**

Let $P_1, P_2 \in \mathbb{R}^2$ and $\theta \in (-\pi, \pi]$. Show that $\mathrm{Rot}_{P_2}(\theta) \circ \mathrm{Rot}_{P_1}(-\theta) = \mathrm{Trans}_{\overrightarrow{QP_2}}$, where $Q = \mathrm{Rot}_{P_1}(\theta)P_2$.

C. **Tilings**

Let $\square$ be a square in $\mathbb{E}^2$ (say with vertices $(1,0),(1,1),(0,1),(0,0)$), and consider the subgroup G of $\mathrm{Eucl}(2)$ generated by the reflections in the 4 sides of $\square$. Prove that the images $g(\square)$ of the given square under $g \in G$ form a tiling of $\mathbb{E}^2$, i.e., the "tiles" $g(\square)$ cover $\mathbb{E}^2$, and tiles overlap only along their boundaries. Prove that G contains the group T of translations in $(2\mathbb{Z})^2$ as a normal subgroup, with quotient $H = G/T$ isomorphic to $\mathbb{Z}/2 \oplus \mathbb{Z}/2$.

D. **The "invariant grid" for motions on** $\mathbb{E}^3$ (part of a past exam question)

Let $h \in \mathrm{Eucl}(3)$ and choose Cartesian coordinates x_1, x_2, x_3 in $\mathbb{R}^3$. For $x \in \mathbb{R}^3$, write $h(x) = (h_1(x), h_2(x), h_3(x))$ and define $\widehat{h}\colon \mathbb{R}^2 \to \mathbb{R}^2$ by

$$(x_1, x_2) \mapsto (h_1(x_1, x_2, 0), h_2(x_1, x_2, 0))$$

Show that $\widehat{h} \in \mathrm{Eucl}(2)$ if and only if $\Pi := \big\{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_3 = 0\big\}$ and $h(\Pi)$ are parallel. [Hint: Given $\widehat{x} = (x_1, x_2), \widehat{y} = (y_1, y_2) \in \mathbb{R}^2$, let $x := (x_1, x_2, 0), y := (y_1, y_2, 0) \in \Pi$. Show $d(\widehat{x}, \widehat{y}) = d(h(x), h(y))$, then calculate $[d(\widehat{x}, \widehat{y})]^2 - [d(\widehat{h}(\widehat{x}), \widehat{h}(\widehat{y}))]^2$, where d denotes the Euclidean distance in $\mathbb{E}^2$ or $\mathbb{E}^3$.]