MA243 Geometry

Worksheet 6

Deadline: Thu 11th Nov 2004, 2:00pm.
Please staple your solutions.

A. **Skew lines**

Let L, M be two skew lines in $\mathbb{A}^3$, and let $P \in \mathbb{A}^3$ be a point not on either L or M. Prove that there is a unique line N through P COPLANAR with L and M, i.e. contained in two planes Π, Π' with $L \subset \Pi$, $M \subset \Pi'$. Deduce that there is at most one line through P meeting L and M, and exactly one unless we're a bit unlucky.

B. **Euclidean motions on** $\mathbb{E}^2$ (part of a past exam question)

Let M, L denote lines in the Euclidean plane which intersect in O at angle ϑ, and let $a \in \mathbb{R}^2$ be along L. Determine the composite $g := \mathrm{Refl}_M \circ \mathrm{Gli}(L, a)$. [Hint: Consider the rectangle $\square ABCD$ or $\square ADCB$ with $\overrightarrow{AD} = \overrightarrow{BC} = a$, centre O, and diagonal $BD = M^{\perp}$. Then determine the pre-images of B and O under g.]

C. **Euclidean motions on** $\mathbb{E}^3$

Express the Euclidean motion $f\colon\ \mathbb{E}^3 \longrightarrow \mathbb{E}^3$ defined by $(x,\, y,\, z) \longmapsto (z+1,\, x+1,\, y+1)$ as TWIST $\mathrm{Trans}_b \circ \mathrm{Rot}_L(\vartheta)$ (see the classification of motions of $\mathbb{E}^3$ in Theorem 3.10).

D. **A nonEuclidean metric**

Let $X = \{A,\, B,\, C,\, D\}$ with $d(A,A) = d(B,B) = d(C,C) = d(D,D) = 0$, $d(A,D) = d(D,A) = 2$, and all the other distances between distinct points equal to 1. Check that d is a metric. Prove that the metric space X is not isometric to any subset of $\mathbb{E}^n$ for any n.

Can you realise X as a subset of a sphere S^2_r of appropriate radius r, with the spherical "great circle" metric?

[Hint: I'm sure you know the riddle: An explorer starts out from base camp, walks 10 miles due South, meets a bear, runs 10 miles due West, then 10 miles due North and finds himself back at base camp. *What colour was the bear?*]