Answers to Worksheet 4

A. If L and M intersect, then $\dim(L \cap M) = 1$ or 0, and by Thm. 1.5 $\dim \langle L, M\rangle = 1$ or 2, giving (a) and (b).

If L and M do not intersect $\langle L, M\rangle$ can only have dimension 2 or 3, with $V_L \cap V_M$ of dimension 1 or 0, respectively. If $\dim(V_L \cap V_M) = 1$, then L, M are parallel, yielding (c). Otherwise they are skew, yielding (d).

B. (a) TRUE

with any $L_1 = L_2$ and L_1', L_2' skew. E.g., in Cartesian coordinates, $L_1' := \{(\lambda,\, 0,\, 0) \mid \lambda \in \mathbb{R}\}$ and $L_2' := \{(\mu,\, \mu,\, 1) \mid \mu \in \mathbb{R}\}$.

(b) FALSE

Counter example: $n = 2$; choose Cartesian coordinates and let $P_0 = Q_0 = (0,\, 0)^T$, $P_1 = Q_1 = (1,\, 0)^T$, $P_2 = Q_2 = (2,\, 0)^T$. The P_i and the Q_i give three pairwise distinct points each. Every $\alpha \in \mathrm{Aff}(2)$ with

$$\alpha \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \qquad \lambda \in \mathbb{R},$$

obeys $\alpha(P_i) = Q_i$ for $i = 0,\, 1,\, 2$. Hence α is not unique.

C. $\mathbf{b} = f(0,0) = (7,7)$, and A has column vectors $f(1,0) - f(0,0)$, $f(0,1) - f(0,0)$, that is, $A = \begin{pmatrix} -5 & -5 \\ -6 & -4 \end{pmatrix}$.

D. P, Q, R are collinear with $\overrightarrow{PR} = \lambda \overrightarrow{PQ}$. The necessary and sufficient condition is that also P', Q', R' are collinear with $\overrightarrow{P'R'} = \lambda \overrightarrow{P'Q'}$. This is necessary because affine maps preserve collinearity and ratio of distances along the same line. It is sufficient because there exists an affine map taking $P \mapsto P'$, $Q \mapsto Q'$, so that if R' is collinear with P', Q' and the ratio is right, we must have $R \mapsto R'$.