Answers to Worksheet 9

A. Linear algebra gives $L_1 \cap L_2 = (8 : 1 : -11)$ and $L_1 \cap L_3 = (0 : 1 : 1)$.

B. (a) TRUE

For any two lines L_1, L_2 in $\mathbb{P}^2_{\mathbb{R}}$, $\dim L_1 = \dim L_2 = 1$. Moreover, $\dim \langle L_1, L_2 \rangle \le 2$. Hence by Thm. 7.7, one has

$$\dim(L_1 \cap L_2) = \dim L_1 + \dim L_2 - \dim \langle L_1, L_2 \rangle \ge 0,$$

proving the claim.

(b) FALSE

For example, the lines $\{(\lambda : \mu : 0 : 0) \mid (\lambda, \mu)^T \in \mathbb{R}^2 - \{0\}\}$ and $\{(0 : 0 : \lambda' : \mu') \mid (\lambda', \mu')^T \in \mathbb{R}^2 - \{0\}\}$ do not intersect in $\mathbb{P}^3_{\mathbb{R}}$.

C. We have $\alpha([x]) = [Ax]$ with $(Ax)_i = \sum_j A_{ij} x_j$. Hence α maps $(1 : \xi_1 : \cdots : \xi_n)$ to $(y_0 : y_1 : \cdots : y_n) = (1 : y_1/y_0 : \cdots : y_n/y_0)$ with $y_i = A_{i,0} + \sum_{j=1}^n A_{i,j}\xi_j$, if $y_0 \ne 0$. Therefore, the induced map on the affine chart U_0 is well-defined if $A_{0,0} + \sum_{j=1}^n A_{0,j}\xi_j \ne 0$, and with respect to the affine coordinates $\xi_1, \ldots, \xi_n$ it is given by

$$(\xi_1, \ldots, \xi_n)^T \longmapsto \left(\frac{A_{1,0} + \sum_{j=1}^n A_{1,j}\xi_j}{A_{0,0} + \sum_{j=1}^n A_{0,j}\xi_j}, \ldots, \frac{A_{n,0} + \sum_{j=1}^n A_{n,j}\xi_j}{A_{0,0} + \sum_{j=1}^n A_{0,j}\xi_j} \right)^T.$$

Using a, b, c, d, we have

$$\xi \longmapsto \frac{a\xi + b}{c\xi + d}.$$

D. $\begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \mu \end{pmatrix}$ with $\lambda, \mu \in \mathbb{R} - \{0\}$.